$$\left(\frac{5}{8}m^4 + 6 + \frac{25}{6}m^3\right) - \left(\frac{8}{3}m^3 + \frac{17}{4}m^4 + \frac{8}{5}m^2\right)$$

Write a polynomial function of least degree that has real coefficients, the following zeros, and a leading coefficient of 1.

$1, \quad -5, \quad -3$

$x = 1 \quad x = -5 \quad x = -3$

$(x-1)(x+5)(x+3) = 0$

$(x-1)(x^2+8x+15)$

$x^3+7x^2+7x-15$

Trigonometric Equations

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Definition

A trigonometric equation is an equation that contains a trig expression with a variable, such as sinx.

Example:

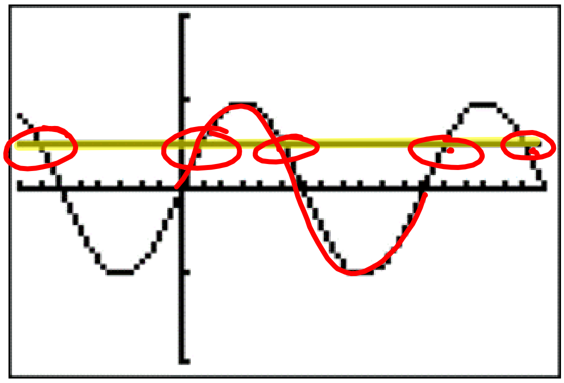
$$\sin x = \frac{1}{2}$$

A solution to this equation is $\frac{\pi}{6}$ because

$$sin\frac{\pi}{6} = \frac{1}{2}$$

Is $\frac{\pi}{6}$ the only solution to this equation?

Consider:



This graph shows 5 different solutions to the equation

$$\sin x = \frac{1}{2}$$

So how can we represent all of the solutions?

Since the period of the function is 2π, first find all of the solutions in $[0, 2\pi)$.

These solutions are $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

Any multiple of 2π can be added to these values and the sine is still $\frac{1}{2}$

So, all solutions can be given by

$$x = \frac{\pi}{6} + 2n\pi \quad or\ x = \frac{5\pi}{6} + 2n\pi$$

with n as any integer

n = # of periods

Find 4 more solutions to $\sin x = \dfrac{1}{2}$

$x = \frac{\pi}{6} + 2n\pi$

$= \frac{\pi}{6} + 2(0)\pi$

$= \boxed{\frac{\pi}{6}}$

$x = \frac{\pi}{6} + 2(1)\pi$

$\frac{\pi}{6} + 2\pi$

$\frac{\pi}{6} + \frac{12\pi}{6}$

$= \boxed{\frac{13\pi}{6}}$

$x = \frac{5\pi}{6} + 2n\pi$

$x = \frac{5\pi}{6} + 2(0)\pi$

$x = \boxed{\frac{5\pi}{6}}$

$x = \frac{5\pi}{6} + 2(1)\pi$

$x = \boxed{\frac{17\pi}{6}}$

Equations involving a Single Trig Function

To solve an equation involving a single trig function:

~Isolate the function on one side of the equation.

~Solve for the variable

Solve:

3 sin x   2 = 5 sin x   1

$-5\sin x \qquad -\cancel{5\sin x}$

$-2\sin x \cancel{-2} = -1$

$+2$

$\frac{\cancel{-2}\sin x}{\cancel{-2}} = \frac{1}{-2}$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

all Sol'n $\begin{cases} x = \frac{7\pi}{6} + 2n\pi \\ x = \frac{11\pi}{6} + 2n\pi \end{cases}$

Solve:

$$5\sin x = 3\sin x + \sqrt{3}$$

$$-3\sin x \quad -3\sin x$$

$$\frac{\cancel{2}\sin x}{\cancel{2}} = \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$[0, 2\pi) \quad x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$* \begin{cases} x = \frac{\pi}{3} + 2n\pi \\ x = \frac{2\pi}{3} + 2n\pi \end{cases}$$

Equations Involving Multiple Angles

$\tan 3x = 1 \quad 0 \le x < 2\pi$

$\tan \theta = 1$

$\frac{\pi}{4}$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{\frac{\pi}{4}}{3} + \frac{n\pi}{3}$

* $x = \frac{\pi}{12} + \frac{\pi}{3} n$

eq. for all sol'n

To find sol'n $0 \le x < 2\pi$

$x = \frac{\pi}{12} + \frac{\pi}{3}(0) = \frac{\pi}{12}$

$x = \frac{\pi}{12} + \frac{\pi}{3}(1) = \frac{5\pi}{12}$

$x = \frac{\pi}{12} + \frac{\pi}{3}(2) = \frac{9\pi}{12}$

$x = \frac{\pi}{12} + \frac{\pi}{3}(3) = \frac{13\pi}{12}$

$x = \frac{\pi}{12} + \frac{\pi}{3}(4) = \frac{17\pi}{12}$

$x = \frac{\pi}{12} + \frac{\pi}{3}(5) = \frac{21\pi}{12} = \frac{7\pi}{4}$

$x = \frac{\pi}{12} + \frac{\pi}{3}(6) = \cancel{\frac{25\pi}{12}}$

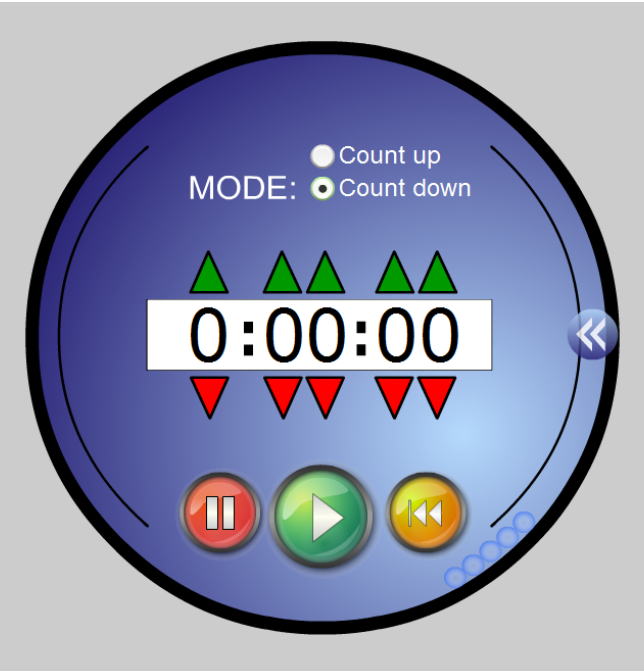
solve:

$\tan 2x = \sqrt{3} \qquad 0 \le x < 2\pi$

$2x = \frac{\pi}{3} + n\pi$

Solve:

$$\sin\frac{x}{2} = -\frac{\sqrt{3}}{2} \qquad 0 \leq x < 2\pi$$



Trig Equations in Quadratic Form

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- Form: $au^2 + bu + x = 0$
  - Where u is a trig function

- Solve by using Quadratic Methods

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  - a. Factor
  - b. Quadratic Formula
  - c. Square Root Method

Solve by factoring:

$$2\cos^2 x + \cos x = 0 \qquad 0 \le x < 2\pi$$

$\cos x(2\cos x + 1) = 0$

① Factor (GCF first)

$\cos x = 0$ $\qquad$ $2\cos x + \cancel{1} = 0$

$\quad -1 \quad -1$

② Set = Zero

$\frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{\cancel{2}\cos x}{\cancel{2}} = \frac{-1}{2}$

③ Solve each eq.

$\cos x = \frac{-1}{2}$

$\frac{2\pi}{3}, \frac{4\pi}{3}$

$$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}}$$

Solve by factoring

$$2\sin^2 x - 3\sin x + 1 = 0 \qquad 0 \le x < 360°$$

$(2\sin x - 1)(\sin x - 1)$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

$30°, 150°$

$\sin x - 1 = 0$

$\sin x = 1$

$90°$

Solve

$$2\sin^2 x = \sin x + 3 \qquad 0 \le x < 2\pi$$

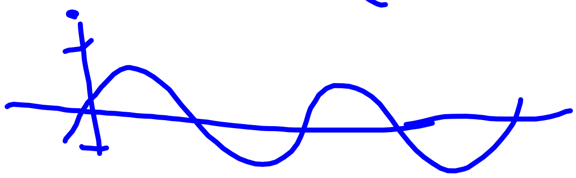
$$2\sin^2 x - \sin x - 3 = 0$$

$$(2\sin x - 3)(\sin x + 1) = 0$$

$$2\sin x - 3 = 0 \qquad \sin x + 1 = 0$$

$$\frac{2\sin x}{2} = \frac{3}{2} \qquad \sin x = -1$$

$$\cancel{\sin x = \frac{3}{2}} \qquad \boxed{\frac{3\pi}{2}}$$



Solve

$$3\cos^2 x - 4\cos x = 0 \qquad 0 \le x < 2\pi$$

$\frac{\pi}{2}, \frac{3\pi}{2}$

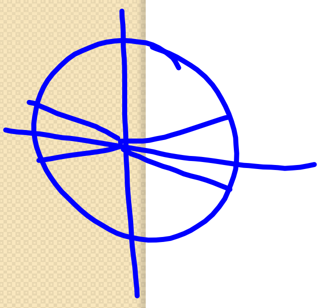
Solve

$$4\sin^2 x - 1 = 0 \qquad 0 \le x < 2\pi$$

$(2\sin + 1)(2\sin - 1)$

$2\sin + 1 = 0 \qquad 2\sin - 1 = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = \frac{1}{2}$



$\frac{7\pi}{6} \qquad \frac{11\pi}{6}$

$\frac{\pi}{6} \qquad \frac{5\pi}{6}$

Solve

$$4\cos^2 x - 3 = 0 \qquad 0 \le x < 360°$$

Separate Two Functions by Factoring

$$\tan x \sin^2 x = 3tanx \qquad 0 \le x < 2\pi$$

$\tan x \sin^2 x - 3\tan x = 0$

$\tan x(\sin^2 x - 3)$

$\tan x = 0 \qquad \sin^2 x - 3 = 0$

$\boxed{0, \pi} \qquad \sqrt{\sin^2} = \sqrt{3}$

$\cancel{\sin \pm \sqrt{3}}$

Separate by Factoring

$$\sin x \tan x = \sin x \qquad 0 \le x < 2\pi$$

$0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$

Solve

$$\tan^2 x \cos x = \tan^2 x \qquad 0 \le x < 360°$$

Solve

$$\cot^2 x \sin x = \cot^2 x \qquad 0 \le x < 2\pi$$

Using Identities to Solve Trig Equations

(All $0 \le x < 2\pi$)

$$2\cos^2 x + 3\sin x = 0$$

$2(1-\sin^2) + 3\sin x$

$\sin^2 + \cos^2 = 1 - \sin^2$

$2 - 2\sin^2 + 3\sin x$

$(-2\sin^2 + 3\sin + 2)$

$2\sin^2 - 3\sin - 2$

$\cancel{(\sin - 2)}(2\sin + 1)$

$\sin x = \frac{-1}{2}$

$\frac{7\pi}{6}$ $\frac{11\pi}{6}$

$$2\sin^2 x - 3\cos x = 0$$

$2(1-\cos^2)-3\cos$

$2-2\cos^2-3\cos$

$-2\cos^2-3\cos+2$

$2\cos^2+3\cos-2$

$(2\cos-1)(\cos+2)$

$\cancel{\cos = -2}$

$\frac{\pi}{3}, \frac{5\pi}{3}$

$$\cos 2x + 3\sin x - 2 = 0$$

$$\cos 2x + \sin x = 0$$

$$2 \cdot \sin x \cos x = \frac{1}{2} \cdot 2$$

$2 \sin \cos = 1$

$2 \sin x \cos x - 1 = 0$

$\sin 2x - 1 = 0$

$\sin 2x = 1$

$\sin \theta = 1$

$\frac{\pi}{2}$

$\theta = \frac{\pi}{2} + 2n\pi$

$\frac{2x}{2} = \frac{\frac{\pi}{2}}{2} + \frac{2n\pi}{2}$

$x = \frac{\pi}{4} + n\pi$

$x = \frac{\pi}{4}, \frac{5\pi}{4} \quad 0 \le x < 2\pi$

$$(\sin x - \cos x)^2 = (1)^2$$

$$(\sin x - \cos x)(\sin x - \cos x) = 1$$

$$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$$

$$1 - 2\sin x\cos x = 1$$

$$-1 \qquad\qquad -1$$

$$-2\sin x\cos x = 0$$